United Kingdom Mathematics Trust

Junior Kangaroo

a member of the Association Kangourou sans Frontières

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Solutions

These are polished solutions and do not illustrate the process of failed ideas and rough work by which candidates may arrive at their own solutions.

It is not intended that these solutions should be thought of as the 'best' possible solutions and the ideas of readers may be equally meritorious.

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1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
B	D	B	C	E	C	C	A	D	E	C	D	E	C	A	D	A	E	C	E	B	D	B	C	A

1. Which single digit should be placed in all three of the boxes shown to give a correct calculation?

$\square\square \times \square = 176$

A 3 B 4 C 5 D 6 E 8

SOLUTION **B**

Note first that $33 \times 3 < 100$ and $55 \times 5 > 250$. However, $44 \times 4 = 176$ and hence the missing digit is 4.

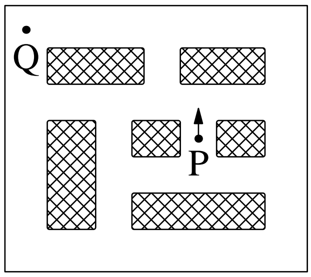
2. The sum of the ages of three children, Ava, Bob and Carlo, is 31. What will the sum of their ages be in three years' time?

A 34 B 37 C 39 D 40 E 43

SOLUTION **D**

In three years' time, each child will be three years older. Hence the sum of their ages will be nine years more than it is at present. Therefore, in three years' time, the sum of their ages will be $31 + 9 = 40$.

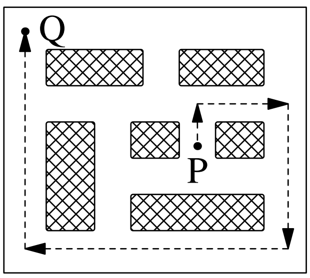
3. Nico is learning to drive. He knows how to turn right but has not yet learned how to turn left. What is the smallest number of right turns he could make to travel from P to Q, moving first in the direction shown?



A 3 B 4 C 6 D 8 E 10

SOLUTION **B**

Since Nico can only turn right, he cannot approach Q from the right as to do so would require a left turn. Therefore he must approach Q from below on the diagram. Hence he will be facing in the same direction as he originally faced. As he can only turn right, he must make a minimum of four right turns to end up facing in the same direction as he started. The route indicated on the diagram below shows that he can reach Q making four right turns. Hence the smallest number of right turns he could make is four.



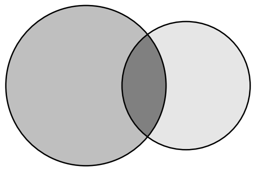
4. A doctor told Mikael to take a pill every 75 minutes. He took his first pill at 11:05. At what time did he take his fourth pill?

A 12:20 B 13:35 C 14:50 D 16:05 E 17:20

Solution **C**

Mikael was told to take a pill every 75 minutes. Therefore he will take his fourth pill 3×75 minutes after he takes his first pill. Now 3×75 minutes is 225 minutes or 3 hours and 45 minutes, He took his first pill at 11:05 and hence he will take his fourth pill at 14:50.

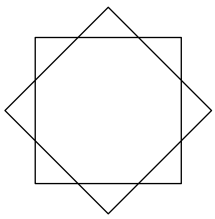
5. When she drew two intersecting circles, as shown, Tatiana divided the space inside the circles into three regions. When drawing two intersecting squares, what is the largest number of regions inside one or both of the squares that Tatiana could create?



A 4 B 6 C 7 D 8 E 9

Solution **E**

Suppose one square has been drawn. This creates one region. Now think about what happens when you draw the second square starting at a point on one side of the first square. One extra region is created each time a side of the second square intersects the first square. Therefore, if there are k points of intersection, there will be k+1 regions when you have finished drawing the second square. However, each side of the second square can intersect at most two sides of the first square. So there can be at most 8 intersection points. Therefore there can be at most 9 regions. The diagram below shows what such an arrangement would look like with 9 regions.



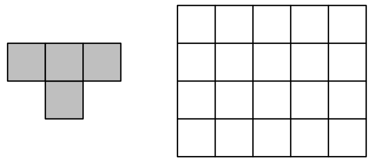
6. The integer 36 is divisible by its units digit. The integer 38 is not.
How many integers between 20 and 30 are divisible by their units digit?

A 2 B 3 C 4 D 5 E 6

Solution **C**

Note that 21 is divisible by 1, 22 is divisible by 2, 24 is divisible by 4 and 25 is divisible by 5. However, 23 , 26, 27, 28 and 29 are not divisible by 3, 6, 7, 8 and 9 respectively. Therefore there are four integers between 20 and 30 that are divisible by their units digit.

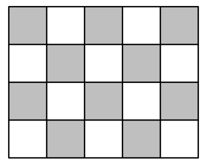
7. What is the largest number of "T" shaped pieces, as shown, that can be placed on the 4×5 grid in the diagram, without any overlap of the pieces?

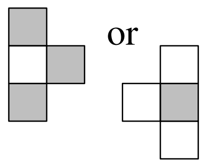


A 2 B 3 C 4 D 5 E 6

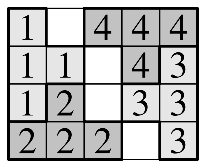
Solution **C**

Colour the cells of the grid alternately black and white, as shown in the first diagram. Each “T” shaped piece fits over three cells of one colour and one of the other colour, as shown in the second diagram. Suppose that for one of the two colours, say white, there are three pieces each covering three cells of that colour. Since there are only ten white cells in the grid, only one white cell is not covered by the three pieces. Hence, as each piece covers at least one white cell, at most one more piece could be placed on the grid. Therefore at most four pieces could be placed on the grid.





Alternatively, if there are no more than two “T” shaped pieces that each cover three white cells and no more than two that cover three black cells, then again there is a maximum of four “T” shaped pieces that could be placed on the grid.



The third diagram shows one of the many different possible ways in which four pieces could be placed, showing that it is possible to place four pieces on the grid.

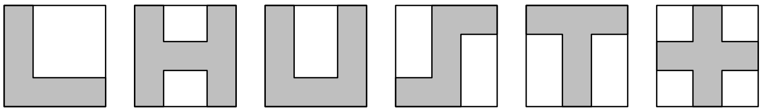
8. Peter the penguin likes catching fish. On Monday, he realised that if he had caught three times as many fish as he actually did, he would have had 24 more fish.
How many fish did Peter catch?

A 12 B 10 C 9 D 8 E 6

Solution **A**

Let the number of fish Peter caught be n. The information in the question tells us that $3n = n + 24$, which has solution $n = 12$. Therefore Peter caught 12 fish.

9. Maria has drawn some shapes on identical square pieces of paper, as shown. Each line she has drawn is parallel to an edge of her paper.



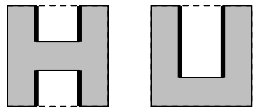
How many of her shapes have the same perimeter as the sheet of paper itself?

A 1 B 2 C 3 D 4 E 5

SOLUTION $\boxed{\textbf{D}}$

In the first, fourth, fifth and sixth diagrams, it is easy to see that the sides of the shapes that do not lie along the sides of the squares have a direct correspondence to the parts of the sides of the squares that are not part of the perimeter of the shapes.

However, in the second and third diagrams, there are some extra sides to the shapes (highlighted in bold) that do not have such a correspondence. Hence the number of shapes with the same perimeter as the square piece of paper is four.



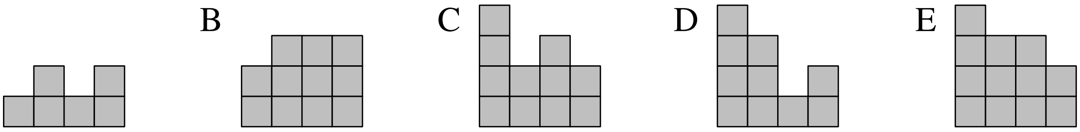
10. Christopher has made a building out of blocks. The grid on the right shows the number of blocks in each part of the building, when viewed from above.

Which of the following gives the view you see when you look at Christopher's building from the front?

4	2	3	2
3	3	1	2
2	1	3	1
1	2	1	2

front

A B C D E



SOLUTION $\boxed{\textbf{E}}$

When you look at Christopher's building from the front, you will see towers of height 4, 3, 3 and 2 as these are the largest numbers of blocks indicated in each of the four columns of the grid. Hence the view that will be seen is E.

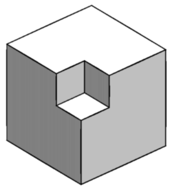
11. In a class election, each of the five candidates got a different number of votes. There were 36 votes cast in total. The winner got 12 votes. The candidate in last place got 4 votes.
How many votes did the candidate in second place get?

A 8 B 9 C 8 or 9 D 9 or 10 E 10

Solution **C**

Let the votes cast for each candidate be 12, x, y, z and 4 with $12 > x > y > z > 4$. Since there were 36 votes cast in total, we have $x + y + z = 20$.
Since $y > z > 4$, and x, y and z are integers, the minimum value of $y + z$ is $6 + 5 = 11$ and hence the maximum value x can be is 9 with an overall solution for (x, y, z) in that case being $(9, 6, 5)$. Also, since $7 + 6 + 5 = 18 < 20$, the minimum value x can take is 8 with an overall solution for (x, y, z) in that case being $(8, 7, 5)$. Hence, although it is not possible to determine exactly how many votes the candidate in second place received, we do know they received either 8 or 9 votes.

12. The diagram shows a wooden cube of side 3 cm with a smaller cube of side 1 cm cut out at one corner. A second cube of side 3 cm has a cube of side 1 cm cut out at each corner.
How many faces does the shape formed from the second cube have?



A 6 B 16 C 24 D 30 E 36

Solution **D**

A standard cube has six faces. When a cube is removed from one corner, the number of faces increases by three, as shown in the diagram in the question. Therefore, the cube with a smaller cube cut out at each of its eight corners has a total of $(6 + 8 \times 3)$ faces. Therefore the second cube has 30 faces.

13. How many pairs of two-digit positive integers have a difference of 50?

A 10 B 20 C 25 D 35 E 40

Solution **E**

The smallest pair of two-digit integers with a difference of 50 is 10 and 60. The largest pair of two-digit integers with a difference of 50 is 49 and 99. Hence there are 40 pairs of two-digit integers with a difference of 50.

14. A lot of goals were scored in a hockey match I watched recently. In the first half, six goals were scored and the away team was leading at half-time. In the second half, the home team scored three goals and won the game. How many goals did the home team score altogether?

A 3 B 4 C 5 D 6 E 9

Solution **C**

Since six goals were scored in the first half and the away side was leading at half-time, the possible half-time scores were 0 - 6, 1 - 5, and 2 - 4. However, we are also told that the home team scored three goals in the second half and won the game. Hence the home team cannot have been more than two goals behind at half-time. Therefore the score at half-time was 2 - 4. Hence the number of goals the home team scored in total was 2 + 3 = 5.

15. In a certain month, the dates of three of the Sundays are prime.
On what day does the 7th of the month fall?

A Thursday B Friday C Saturday D Monday E Tuesday

Solution **A**

The diagram below shows a calendar for a month with dates that are prime shown in bold.

A	B	C	D	E	F	G
1	**2**	**3**	4	**5**	6	**7**
8	9	10	**11**	12	**13**	14
15	16	**17**	18	**19**	20	21
22	**23**	24	25	26	27	28
29	30	**31**				

It can be seen that the only column with three dates shown in bold is Column C. Therefore if a month has three Sundays with dates that are prime, that month has 31 days in it and the dates of the Sundays are the 3rd, 10th, 17th, 24th and 31st of that month. If the 3rd is a Sunday, then the 4th is a Monday, the 5th is a Tuesday and the 6th a Wednesday. Hence the 7th of the month is a Thursday.

16. Alisha wrote an integer in each square of a 4×4 grid. Integers in squares with a common edge differed by 1. She wrote a 3 in the top left corner, as shown. She also wrote a 9 somewhere in the grid. How many different integers did she write?

3			

A 4 B 5 C 6 D 7 E 8

SOLUTION **D**

Since the integers in squares with a common edge differ by 1, the integers in the two squares with a common edge to the square with a 3 in are either 2 or 4. Hence they are both ≤ 4. Similarly, if the integer in a square is ≤ 4, then the integers in the squares with a common edge to that square are ≤ 5, and so on. This gives a set of inequalities for the integers in all the squares, as shown in Figure 1.

3	≤ 4	≤ 5	≤ 6
≤ 4	≤ 5	≤ 6	≤ 7
≤ 5	≤ 6	≤ 7	≤ 8
≤ 6	≤ 7	≤ 8	≤ 9

Fig. 1

Since only one square could contain an integer as big as 9 and we are told that Alisha wrote a 9 somewhere in the grid, the 9 must be in the bottom right corner. The integers in the squares with a common edge to the square with a 9 in are either 8 or 10. Hence they are both ≥ 8. We can continue this process in a similar way to obtain a second set of inequalities for the integers in all the squares, as shown in Figure 2.

3	≥ 4	≥ 5	≥ 6
≥ 4	≥ 5	≥ 6	≥ 7
≥ 5	≥ 6	≥ 7	≥ 8
≥ 6	≥ 7	≥ 8	**9**

Fig. 2

An integer that is both $\leq n$ and $\geq n$ must be n. Therefore, from the inequalities given for the integers in all the squares in Figures 1 and 2, we can deduce that the integers Alisha wrote were as shown in Figure 3. From this we see that Alisha wrote only the integers 3, 4, 5, 6, 7, 8 and 9 in the grid. Hence she wrote seven different integers in total.

3	**4**	**5**	**6**
4	**5**	**6**	**7**
5	**6**	**7**	**8**
6	**7**	**8**	**9**

Fig. 3

17. Ali, Bev and Chaz never tell the truth. Each of them owns exactly one coloured stone that is either red or green. Ali says, "My stone is the same colour as Bev's". Bev says, "My stone is the same colour as Chaz's". Chaz says, "Exactly two of us own red stones". Which of the following statements is true?

A Ali's stone is green
B Bev's stone is green
C Chaz's stone is red
D Ali's stone and Chaz's stone are different colours
E None of the statements A to D are true

Solution **A**

The question tells us that none of the three people tell the truth.
Ali says that his stone is the same colour as Bev's and so we can deduce that Ali and Bev own different coloured stones and hence that there is at least one stone of each colour.
Bev says that her stone is the same colour as Chaz's and so we can deduce that Bev and Chaz own different coloured stones and also that Ali and Chaz own the same coloured stones.
Chaz says that exactly two of the stones are red and so we can deduce that Chaz and Ali own green stones and that Bev owns a red stone.
Hence only statement A is correct.

18. There are 66 cats in my street. I don't like 21 of them because they catch mice. Of the rest, 32 have stripes and 27 have one black ear. The number of cats with both stripes and one black ear is as small as it could possibly be. How many cats have both stripes and one black ear?

A 5 B 8 C 11 D 13 E 14

Solution **E**

The information in the question tells us that the number of cats in my street that don't catch mice is $66 - 21 = 45$. Of these 45, let the number of cats with both stripes and one black ear be X and let the number of cats with neither stripes nor one black ear be Y. Since 32 cats have stripes and 27 have one black ear, we have $32 + 27 - X + Y = 45$ and hence that $14 + Y = X$. We are told that the number of cats with both stripes and one black ear is as small as possible and hence that number is 14 with no cats having neither stripes nor one black ear.

19. A group of 40 boys and 28 girls stand hand in hand in a circle facing inwards. Exactly 18 of the boys give their right hand to a girl. How many boys give their left hand to a girl?

A 12 B 14 C 18 D 20 E 22

Solution **C**

We are told that 18 boys give their right hand to a girl and that there are 40 boys in the circle in total. Therefore 22 boys give their right hand to a boy. Since all the children are facing inwards, this means that all the boys who have a boy giving them their right hand, will in return be giving a boy their left hand. Hence 22 boys give their left hand to a boy. Therefore there will be 18 boys who give their left hand to a girl.

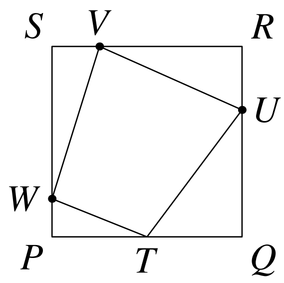
20. For how many three-digit numbers can you subtract 297 and obtain a second three-digit number which is the original three-digit number reversed?

A 5 B 10 C 20 D 40 E 60

Solution **E**

Suppose that 'pqr' is a three-digit number whose digits are reversed when 297 is subtracted. Since 'pqr' represents the number $100p + 10q + r$ and 'rqp' represents $100r + 10q + p$, we have $100p + 10q + r - 297 = 100r + 10q + p$. This equation can be rearranged to give $99p - 99r = 297$ and hence we have $p - r = 3$. Since we know that 'rqp' is a three-digit number, $r \neq 0$. Therefore there are six possibilities for the pair (p, r), namely (4, 1), (5, 2), (6, 3), (7, 4), (8, 5) and (9, 6). The middle digit, q, can be any one of the 10 digits. Therefore the number of possible values for the original three-digit number is $6 \times 10 = 60$.

21. The diagram shows a square $PQRS$ with area $120\,\text{cm}^2$. Point T is the mid-point of PQ. The ratio $QU : UR = 2 : 1$, the ratio $RV : VS = 3 : 1$ and the ratio $SW : WP = 4 : 1$.
What is the area, in cm^2, of quadrilateral $TUVW$?



A 66 B 67 C 68 D 69 E 70

SOLUTION **B**

We are told that T is the mid-point of PQ. Hence $PT = \frac{1}{2}PQ$. Similarly, we are told that $SW : WP = 4 : 1$. Therefore $WP = \frac{1}{5}PS$. Hence $\frac{1}{2}(PT \times PW) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{5}(PQ \times PS)$. It follows that the fraction of the area of square $PQRS$ that lies in triangle PTW is $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{5}$ or $\frac{1}{20}$. Similarly, the fraction of the square that lies in triangle TQU is $\frac{1}{2} \times \frac{1}{2} \times \frac{2}{3}$ or $\frac{1}{6}$, the fraction of the square that lies in triangle URV is $\frac{1}{2} \times \frac{1}{3} \times \frac{3}{4}$ or $\frac{1}{8}$ and the fraction of the square that lies in triangle VSW is $\frac{1}{2} \times \frac{1}{4} \times \frac{4}{5}$ or $\frac{1}{10}$. Therefore the fraction of the square that lies outside quadrilateral $TUVW$ is $\frac{1}{20} + \frac{1}{6} + \frac{1}{8} + \frac{1}{10} = \frac{53}{120}$. Since we are told in the question that the area of square $PQRS$ is $120\,\text{cm}^2$, the area of quadrilateral $TUVW$, in cm^2, is $(1 - \frac{53}{120}) \times 120 = 67$.

22. In the Maths Premier League, teams get 3 points for a win, 1 point for a draw and 0 points for a loss. Last year, my team played 38 games and got 80 points. We won more than twice the number of games we drew and more than five times the number of games we lost.
How many games did we draw?

A 8 B 9 C 10 D 11 E 14

SOLUTION **D**

Let the number of matches my team won, drew and lost be x, y and z respectively. Since we played 38 games in total and got 80 points, $x + y + z = 38$ and $3x + y = 80$. The information in the question tells us that $x > 2y$ and $x > 5z$. Since $3 \times 27 = 81 > 80$, it follows that $x < 27$. The possible values for x, y and z which satisfy the two equations are (26, 2, 10), (25, 5, 8), (24, 8, 6), (23, 11, 4), (22, 14, 2) and (21, 17, 0). The only one of these combinations which also satisfies the two inequalities is (23, 11, 4) and hence my team drew 11 matches.

23. For a given list of three numbers, the operation "changesum" replaces each number in the list with the sum of the other two. For example, applying "changesum" to 3, 11, 7 gives 18, 10, 14. Arav starts with the list 20, 2, 3 and applies the operation "changesum" 2023 times.
What is the largest difference between two of the three numbers in his final list?

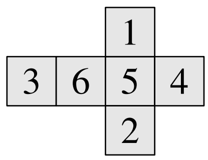
A 17 B 18 C 20 D 2021 E 2023

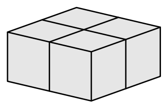
SOLUTION **B**

Let the three numbers in the list be X, Y and Z, where we will assume that $X \geq Y \geq Z$. The differences then can be written as $X - Y$, $X - Z$ and $Y - Z$. After the operation "changesum" has been applied to the list, the values in the new list become $Y + Z$, $X + Z$ and $X + Y$. The differences between these new values are $(X + Z) - (Y + Z)$, $(X + Y) - (Y + Z)$ and $(X + Y) - (X + Z)$ which are equal to $X - Y$, $X - Z$ and $Y - Z$.

Therefore it can be seen that the differences between the numbers in the list after applying the operation "changesum" are the same as the differences between the numbers in the list before applying the operation "changesum". Hence, the largest difference between two numbers in Arav's list after applying "changesum" 2023 times will be equal to the largest difference between two numbers in the original list, that is $20 - 2$, which is equal to 18.

24. Emily makes four identical numbered cubes using the net shown. She then glues them together so that only faces with the same number on are glued together to form the $2 \times 2 \times 1$ block shown.
What is the largest possible total of all the numbers on the faces of the block that Emily could achieve?





A 72 B 70 C 68 D 66 E 64

SOLUTION **C**

Each cube in the block has two adjacent faces that do not form part of the faces of the block. Since the numbers 1 and 2 are on opposite faces, it is not possible for both of these numbers to be hidden. However, the numbers 1 and 3 are on adjacent faces. Therefore, to obtain the largest possible total on the faces of the block, each cube will have numbers 1 and 3 hidden. Hence the largest possible total of the numbers on the faces on the block is $4 \times (6 + 5 + 4 + 2) = 68$.

25. Tony had a large number of 1p, 5p, 10p and 20p coins in a bag. He removed some of the coins. The mean value of the coins he removed was 13p. He noticed that a 1p piece in his group of removed coins was damaged so he threw it away. The mean value of the rest of his removed coins was then 14p.
How many 10p coins did he remove from the bag?

A 0 B 1 C 2 D 3 E 4

SOLUTION **A**

Let the number of coins Tony removed be n and let the total value of these coins, in pence, be X. The initial information in the question tells us that $\frac{X}{n} = 13$. Also, since when he threw away a 1p coin, the mean value of his coins increased to 14, we have $\frac{X-1}{n-1} = 14$. Therefore $X = 13n$ and $X - 1 = 14(n - 1)$ and hence $13n - 1 = 14n - 14$, which has solution $n = 13$. Therefore Tony removed 13 coins with a total value of 13×13p, or 169p.

We have found that the total of the 13 coins Tony removed is 169p. It is impossible to have a total of 169p with only 5p, 10p and 20p coins as that would give a total that is a multiple of 5. Hence Tony must have removed either four 1p coins or nine 1p coins. However, the latter is impossible as it would mean that the remaining four coins would need to have a total value of 160p and the maximum value from four 5p, 10p and 20p coins is only $4 \times 20\text{p} = 80\text{p}$. Therefore Tony removed four 1p coins. This means that the total value of the other nine coins is 165p. Now, since $9 \times 20\text{p} = 180\text{p}$, which is too large and $7 \times 20\text{p} + 2 \times 10\text{p} = 160\text{p}$, which is too small, Tony must have removed eight 20p coins plus an additional one 5p coin to have the required total. Therefore Tony did not remove any 10p coins.